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Abstracts of Papers to Appear

On the Smoothness Constraints for Four-Dimensional Data Assimilation. Ching-Long Lin,* Tianfeng Chai,† and Juanzhen Sun.† *Department of Mechanical Engineering and IIHR Hydroscience & Engineering, The University of Iowa, Iowa City, Iowa 52242-1527; and †National Center for Atmospheric Research, P.O. Box 3000, Boulder, Colorado 80307-3000.

An algorithm for determination of the weights of smoothness penalty constraints for the four-dimensional variational data assimilation technique is proposed and evaluated. To study the nature of smoothness penalty constraints, a simple nonlinear harmonic oscillator problem is first considered. Penalizing smoothness constraints is found to make the modified Hessian matrix of the cost function more positive definite, akin to the idea behind the modified line search Newton's methods. However, the use of the derivative smoothness constraints with a fixed coefficient does not warrant uniform imposition of these constraints at every iteration. A remedy is to control the ratio of the smoothness penalty function over the cost function, which can dramatically increase the positive definite area. On the other hand, the large smoothness coefficients that resulted from this approach can deteriorate the convergence property of the minimization problem. Based on these observations, an algorithm for tuning the weights of smoothness constraints is proposed to overcome the aforementioned problems. The algorithm is first applied to a simple dynamic problem. It is then tested on the retrieval of microscale turbulent structures in a simulated convective boundary layer. This method is further evaluated on the retrieval of a strong meso-scale thunderstorm outflow from Doppler radar data. The results show that the algorithm yields efficient retrieval.

Least-Squares Spectral Elements Applied to the Stokes Problem. M. M. J. Proot and M. I. Gerrtisma. Delft University of Technology, Aerospace Engineering, Section Aerodynamics, Kluyverweg 1, Delft, The Netherlands.

Least-squares spectral element methods are based on two important and successful numerical methods: spectral/*hp* element methods and least-squares finite element methods. In this respect, least-squares spectral element methods seem very powerful since they combine the generality of finite element methods with the accuracy of the spectral methods and also have the theoretical and computational advantages of the least-squares methods. These features make the proposed method a competitive candidate for the solution of large-scale problems arising in scientific computing. In order to demonstrate its competitiveness, the method has been applied to an analytical problem and the theoretical optimal and suboptimal a priori estimates have been confirmed for various boundary conditions. Moreover, the exponential convergence rates, typical for a spectral element discretization, have also been confirmed. The comparison with the classical Galerkin spectral element method revealed that the least-squares spectral element method is as accurate as the Galerkin method for the smooth model problem.

Highly Energy-Conservative Finite Difference Method for a Cylindrical Coordinate System. Koji Fukagata*,† and Nobuhide Kasagi.* *Department of Mechanical Engineering, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan; and †Institute for Energy Utilization, AIST, 1-2-1 Namiki, Tsukuba-shi, Ibaraki 305-8564, Japan.

A highly energy-conservative second-order-accurate finite difference method for the cylindrical coordinate system is developed. It is rigorously proved that energy conservation in discretized space is satisfied when appropriate interpolation schemes are used. This argument holds not only for an unequally spaced mesh but also for an equally spaced mesh on cylindrical coordinates but not on Cartesian coordinates. Numerical tests are undertaken for an inviscid flow with various schemes, and it turns out that the proposed scheme offers a superior energy-conservation

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property and greater stability than the intuitive and previously proposed methods, for both equally spaced and unequally spaced meshes.

Nodal High-Order Discontinuous Galerkin Methods for the Spherical Shallow Water Equations. F. X. Giraldo,* J. S. Hesthaven,† and T. Warburton.‡ *Naval Research Laboratory, Monterey, California 93943; †Division of Applied Mathematics, Brown University, Providence, Rhode Island 02912; and ‡Department of Mathematics and Statistics, University of New Mexico, Albuquerque, New Mexico 87131.

We present a high-order discontinuous Galerkin method for the solution of the shallow water equations on the sphere. To overcome well-known problems with polar singularities, we consider the shallow water equations in Cartesian coordinates, augmented with a Lagrange multiplier to ensure that fluid particles are constrained to the spherical surface. The global solutions are represented by a collection of curvilinear quadrilaterals from an icosahedral grid. On each of these elements the local solutions are assumed to be well approximated by a high-order nodal Lagrange polynomial, constructed from a tensor-product of the Legendre–Gauss–Lobatto points, which also supplies a high-order quadrature. The shallow water equations are satisfied in a local discontinuous element fashion with solution continuity being enforced weakly. The numerical experiments, involving a comparison of weak and strong conservation forms and the impact of over-integration and filtering, confirm the expected high-order accuracy and the potential for using such highly parallel formulations in numerical weather prediction.

A Moving Mesh Method for the Solution of the One-Dimensional Phase-Field Equations. J. A. Mackenzie and M. L. Robertson. Department of Mathematics, University of Strathclyde, Livingstone Tower, 26 Richmond Street, Glasgow, G1 1XH, Scotland.

A moving mesh method is developed for the numerical solution of one-dimensional phase-change problems modelled by the phase-field equations. The computational mesh is obtained by equidistribution of a monitor function tailored for the functional variation of the phase field in the interfacial region. Existence and uniqueness of the discretised equations using a moving mesh are also established. Numerical results are given for classical and modified Stefan test problems. The numerical algorithm is relatively simple and is shown to be far more efficient than fixed grid methods.